**ENGSCI CT2 2023 Lab 2 Univariate Minimisation Answer Worksheet**

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**Question 1:** Your answer…

|  |  |
| --- | --- |
|  |  |
| a + tau\*(b - a) | a + (1 - tau) \* (b - a) |

**Question 2:** Your answer…

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  | (x’k, x’’k, bk ) | (ak, x’’k, x’k) |
|  | (ak, x’k, x’’) | (x’’k, x’k, bk) |

**Question 3: Code submission of golden.py**

**Question 4: GS & quadratic-only Brent’s method on f0, f1 starting from [a,b]=[0,3]**

*Using the plots and logging output from task1.py, comment on the performance of both the golden section method and the quadratic-only Brent’s method (as provided in brent.py), and discuss how their performance is affected by the function being minimized. Briefly discuss reasons for the behaviours you observe.*

Answer the following by completing the tables below*: “comment on the performance of both the golden section method and the quadratic-only Brent’s method (as provided in brent.py), “*

*Golden Section Method:*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Experiment | Function | Minimum for | Starting Interval of Uncertainty | ? | Method converges? (Y/N) | Method converges to minimum? (Y/N) | If method did not converge to minimum, explain what happened and why. (If the answer is the same as another answer, just say “Same as C2” for example.) |
| A0 |  | 2 | [0,3] | Y | Y | Y |  |
| A1 |  | 2 | [0,3] | Y | Y | Y |  |

*Quadratic-only Brents Method:*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Experiment | Function | Minimum for | Starting Interval of Uncertainty | ? | Method converges? (Y/N) | Method converges to minimum? (Y/N) | If method did not converge to minimum, explain what happened and why. (If the answer is the same as another answer, just say “Same as C2” for example.) |
| A0 |  | 2 | [0,3] | Y | Y | Y |  |
| A1 |  | 2 | [0,3] | Y | Y | Y |  |

*“…discuss how their performance is affected by the function being minimized. “*

Your Answer:

* As both functions (f0­ and f1) are well-behaved over the region [0,3], this means there is only one local minimum within the region. Because of this, both methods will yield correct results by reducing the feasible region down and always including the one minimum point, until it very narrowly confines it. Both of them converge fairly quickly (after 10-17 iterations) to the correct minimum point.

*“Briefly discuss reasons for the behaviours you observe.”*

Your Answer:

* This behaviour is observed because, as mentioned above, both graphs are well behaved on the provided domain. Because the golden section search does not make any explicit assumptions, and the Brent assumption that the graph is locally quadratic are both satisfied, the width interval decreases towards 0 for both methods.

**Question 5: GS & quadratic-only Brent’s method on f0, f1 starting from [a,b]=[0,1]**

*Using these plots, and logging output from your code, carefully explain and contrast*

*what has happened for each of the two functions for each method.*

Complete the tables below to answer the following: *Using these plots, and logging output from your code, carefully explain ~~and contrast~~ what has happened for each of the two functions for each method.*”

*Golden Section Method:*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Experiment | Function | Minimum for | Starting Interval of Uncertainty | ? | Method converges? (Y/N) | Method converges to minimum? (Y/N) | If method did not converge to minimum, explain what happened and why. (If the answer is the same as another answer, just say “Same as C2” for example.) |
| B0 |  | 2 | [0,1] | N | Y | N | The minimum is not within the given domain, so it approaches it but cannot reach it as the method is confined by the domain. |
| B1 |  | 2 | [0,1] | N | Y | N | The minimum is not within the given domain, so it approaches it but cannot reach it as the method is confined by the domain. |

*Quadratic-only Brents Method:*

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Experiment | Function | Minimum for | Starting Interval of Uncertainty | ? | Method converges? (Y/N) | Method converges to minimum? (Y/N) | If method did not converge to minimum, explain what happened and why. (If the answer is the same as another answer, just say “Same as C2” for example.) |
| B0 |  | 2 | [0,1] | N | Y | Y |  |
| B1 |  | 2 | [0,1] | N | N | N | The minimum is not within the given domain, and the graph is concave down, so it finds maxima instead of minima, leading to the width of the interval not decreasing lower than ~0.65. |

“*Using these plots, and logging output from your code, carefully ~~explain and~~ contrast*

*what has happened for each of the two functions for each method.*”

Your Answer:

* Using the golden section search, both graphs undergo the same process, where the method tries to minimise over the domain [0,1]. However as the functions are not well-behaved over this domain, there is no singular minimum to converge to. The minimum is to the right of the domain, so the method (in both cases) pushes as far to the right as possible within the given domain, and results in choosing the point at x=1.
* Using the Jarrat (quadratic only) method, it does not have to remain within the given domain, so for f0 this method is able to escape the domain [0,1] and find the correct minimum point at x=2. It can do this because the quadratic that gets fit is not constrained to the [0,1] domain, so can find new edge values outside of this region, and can encompass the correct value this way.
* The Brent method assumes that when finding the stationary point of the parabola, that it will be a minimum. However, for f1, the left side of the graph is concave down (not concave up), so the stationary point that is found is a maximum. This violates the assumptions of the model so it will not converge. This leads to the width of the search region not approaching zero, instead getting stuck at around 0.65 units wide.

**Question 6: GS + Quadratic-only Brent’s on x2, x4, x6, x8**

*Using the plots and output from task3.py, comment on and contrast the performance of the golden section method and quadratic-only Brent’s method on these functions. Give brief explanations for the key differences and trends you observe in the runs.*

Answer the following by completing the tables below:

*Using the plots and output from task3.py, comment on ~~and contrast~~ the performance of the golden section method and quadratic-only Brent’s method on these functions.*

Golden Section Method

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Experiment | Function | Minimum for | Starting Interval of Uncertainty | ? | Method converges? (Y/N) | Method converges to minimum? (Y/N) | If method did not converge to minimum, explain what happened and why. (If the answer is the same as another answer, just say “Same as C2” for example.) |
| C2 |  | 0 | [-0.3,1] | Y | Y | Y |  |
| C3 |  | 0 | [-0.3,1] | Y | Y | Y |  |
| C4 |  | 0 | [-0.3,1] | Y | Y | Y |  |
| C5 |  | 0 | [-0.3,1] | Y | Y | Y |  |

Quadratic-only Brent’s Method

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Experiment | Function | Minimum for | Starting Interval of Uncertainty | ? | Method converges? (Y/N) | Method converges to minimum? (Y/N) | If method did not converge to minimum, explain what happened and why. (If the answer is the same as another answer, just say “Same as C2” for example.) |
| C2 |  | 0 | [-0.3,1] | Y | Y | Y |  |
| C3 |  | 0 | [-0.3,1] | Y | Y | Y |  |
| C4 |  | 0 | [-0.3,1] | Y | N | N | The quadratic method encounters a divide-by-zero error so cannot continue the process, and does not converge. |
| C5 |  | 0 | [-0.3,1] | Y | N | N | Same as ^above^ |

*Using the plots and output from task3.py~~, comment on~~ and contrastthe performance of the golden section method and quadratic-only Brent’s method on these functions.*

Your answer:

* *For the golden search method, As these functions all only have the one minimum at x=0, and are all concave-up, there should be no issue for the golden section. In regards to the performance, the Golden section search converges after 16 evaluations for each function, for the given tolerance.*
* *Comparatively, the quadratic only Brent’s method converges after only 6 evaluations for the x^2 graph (f2). This makes sense because the graph is purely quadratic so the quadratic search method should converge very quickly. F3 = x^4 is similar and converges quickly after only 10 evaluations. However, for f4 and f5, there is a problem. Within the quadratic-only brent method, we must calculate the x4 point using an operation involving a denominator, and its possible for this denominator to be 0. These two functions lead to a divide-by-zero problem before they can converge, so are unable to converge to the desired tolerance for our width-interval. This makes the golden section better for these functions, as it is does not encounter this issue.*

*Give brief explanations for the key differences and trends you observe in the runs.*

Your answer:

* We can observe from the graphs outputted by the code that the Brent search converges much faster for f2 and f3. We can see far more functions evaluations at places far from the minimum point in golden search, indicating that it took more function evaluations, therefore was less efficient than the Brent search counterpart. For f4 and f5, it is also likely that the brent method would have been faster if it did not encounter the divide-by-zero issue, however the option with better performance for these graphs is definitely the Golden Section search method (It is more reliable).

**Question 7: Code submission of the FullBrent code**

**Question 8:** *Using the plots and log output, compare and contrast the performance of the FullBrent implementation on f2 and f5, giving an explanation for the observed differences. Be sure to look carefully at the log output.*

* *For f2, which is f(x)=x^2 , the FullBrent code is very efficient at finding the correct minimum. It converges after only 12 function evaluations, while f5 takes much longer to converge, taking 63 function evaluations. The reason for this is because f2(x) = x^2 is purely quadratic, so the quadratic steps taken by the full brent implementation help is to converge very quickly. Comparatively, the f5(x)=x^8 is not quadratic, so the quadratic steps taken by the fullbrent algorithm don’t cause it to converge quite so quickly. Also, for f5, the method encounters far more denominators that are close to zero, so has more occasions when it reverts to the golden section instead of the quadratic search, which makes the process less efficient. So overall, the fullbrent algorithm far more efficient on f2 than f5.*